1. Prove, using the Pumping Lemma, that \( \{a^nba^{2n} \mid n > 0 \} \) is not regular.

2. Let \( L = \{ wa^n \mid w \in \{a,b,c\}^* \} \). In other words, \( L \) consists of words \( wa^n \) where \( w \) contains \( a \)'s, \( b \)'s, and \( c \)'s and \( n \) is the length of \( w \).
   
   i. Use the Myhill-Nerode Theorem to prove that \( L \) is not regular.
   
   ii. Use the Pumping Lemma to prove \( L \) is not regular.

3. Let \( L = \{ a^n \mid n \text{ is not a prime number} \} \).

   i. Prove that \( L \) is not regular.

   ii. Prove that \( L \) satisfies the Pumping Lemma.

4. Give an example of a regular language \( R \) and a non-regular language \( L \) such that \( R + L \) is regular, and prove or justify that \( R + L \) is regular.

5. Give an example of a regular language \( R \) and a non-regular language \( L \) such that \( R + L \) is non-regular, and prove or justify that \( R + L \) is non-regular.

6. Let \( L \) be a regular language over \( \Sigma = \{a,b\} \). Define \( L' = \{ x \mid \text{there exists } y \in \Sigma^* \text{ such that } xy \in L \} \). Is \( L' \) regular? Either prove it is or give an example to show it may not always be.